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Improved Definition of
Crustal Anomalies for Magsat Data
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During the sixth quarter of this project, the DST indices have been compiled for the time period that Magsat was in orbit, and magnetic tapes containing these indices have been delivered to both the data center and the Magsat project manager.

As an alternative method for verifying the DST correctional scheme proposed earlier in this investigation, another approach has been developed for separating the portions of the magnetic field measured by Magsat that arise from internal and external sources.

In this approach we begin by considering a set of functions which satisfy Dirichlet's problem for the circle:

$$u\,(r,\theta) = 1/2\ a_o + \sum_{n=1}^{N} \left(\frac{r}{r_o}\right)^{n} (a_n \cos n\theta + b_n \sin n\theta)$$

where the source is inside the circle, and

$$v\,(r,\theta) = \sum_{n=1}^{N} \left(\frac{r_o}{r}\right)^{n+1} (c_n \cos n\theta + d_n \sin n\theta)$$

for sources outside the circle.

These functions are harmonic and tell us that if we know the value of a potential function on a circular boundary, we know its value everywhere inside and outside the circle. The two equations describe the cases where the sources are internal and external.

In the case of the Magsat data, measurements are made both above and below a circular boundary due to the ellipticity of the satellites orbit. Therefore, implicit in these measurements is information on the attenuation of each frequency in the magnetic field for a particular orbit as a function of altitude. Adescription is given here of an attempt to separate the internal and external fields measured by an orbiting satellite.

A particular measurement of the earths magnetic field at a distance of r kilometers from the center of the earth can be expressed as

$$v = u + v = 1/2\ a_o + \sum_{n+1}^{N} \left(\frac{r}{r_o}\right)^{n} (a_n \cos n\theta + b_n \sin n\theta) + \sum_{n=1}^{N} \left(\frac{r_o}{r}\right)^{n+1} (c_n \cos n\theta + d_n \sin$$

To test this method, a set of sample coefficients were used to compute field values along a simulated satellite orbit. This data was then used to try to recover the original coefficients used to generate the field values. The satellite orbit was given by r = 6800 + 100 sinθ where θ is the central angle of the orbit and max and min values of r were 6700 km and 6900 km with 6800 km the mean value. To compute the field 6 harmonics were used for the inner terms and six for the outer. No constant term was used for the input but one was computed for the output. The input coefficients were solved for in two ways:

(1) Inversion of a 25 x 25 matrix

(2) A modified version of the least squares solution which utilized the matrix inversion lemma, to obtain the recursive least squares solution. For this case, 2048 points were used for input. Note that in using this method it is not necessary to invert a matrix.

Table 1 shows the results of solving for input coefficients using r(int) = r(ext) = 6800 km. It can be seen that the matrix inversion method gives the correct values for the 6th harmonic, but the lower order terms become increasingly bad. The recursive least squares method doesn't do as well, but is in the right ball park for the sixth harmonic.

In tables 2 through 4, the same input data was used, but the radii of the circles on which the internal and external boundaries were defined was varied. In table 2, r(int) = 6400, (approximately 1 earth radii) and r(ext) = 5 x 6400 (approximately 5 earth radii). In the matrix inversion solution the external terms blow up, while only the 5th and 6th terms are approximately right for the internal coefficients. The recursive least squares solution gets all the internal terms correct (which are all that really contribute anything to the field at the satellite orbit in this case) but the external terms never really get off the ground. This seems to be a peculiar property of the recursive least squares method. It is probably due to the fact that the very small attenuation coefficients for the external terms tend to suppress their contribution completely.

The values in table 3 used 6400 for r(int) and 7200 for r(ext). Thus the internal and external generating boundaries were approximately equidistant from the satellite orbit. Again, the matrix inversion was only right for the higher order terms, while the recursive least squares seemed to split up the coefficients approximately evenly between internal and external terms.

If we take a series of 4N+1 measurements at equal angular increments through the course of one orbit, y_i ($i=1,2,\ldots,4N+1$), we get 4N+1 equations and an equal number of unknowns. These equations can be written in matrix notation as where,

$$\bar{y} = M\bar{x}$$

$$\bar{y} = (y_o, y_1, \ldots, y_{4n})^T$$

$$\bar{x} = (a_o, a_1, b_1, \ldots, c_{4n}, d_{4n})^T$$

and

$$M = \begin{vmatrix} \frac{r_1}{r_o}\cos\theta & \frac{r_1}{r_o}\sin\theta & \cdots\cdots & \frac{r_o}{r_1}^{n+1}\cos n\theta & \frac{r_o}{r_1}^{n+1}\sin n\theta \\ \frac{r_2}{r_o}\cos\theta & \frac{r_2}{r_o}\sin\theta & \cdots\cdots & \frac{r_o}{r_2}^{n+1}\cos n\theta & \frac{r_o}{r_2}^{n+1}\sin n\theta \\ \vdots & \vdots & & \vdots & \vdots \\ \frac{r_{4n}}{r_o}\cos\theta & \frac{r_{4n}}{r_o}\sin n\theta & \cdots\cdots & \frac{r_o}{r_{un}}^{n+1}\cos n\theta & \frac{r_o}{r_{un}}^{n+1}\sin n\theta \end{vmatrix}$$

If there are more measurements than unknowns, a least squares solution can be obtained using the relation

$$\hat{x} = (M^T M)^{-1} M^T Y$$

where y is an mx1 column vector and $m>4N+1$.

It should be noted that this method is similar to that used by Henderson & Cordell in adjusting potential field data to a common altitude. The difference being that here the data is considered for the problem of the circled rather than for a straight line or plane; i.e., the attenuation factor is of the form $(\frac{r}{r_o})^n$ rather than e^{-hn}. In any case, this method can be used to adjust satellite data to a common altitude, and, if it is applied to component data rather than total field data, the assumption of harmonicity is valid.

Table 4 used r(int) = 6400 km and r(ext) = 2 x r(int)

Table 5 was the same with different input coefficients.

In table 6 r(int)=r(ext)=6800, as in the first case. However, here the equation for the satellite orbit was changed to

$$r = 6800 + 3200 \times \sin \theta$$

In this case, the last 4 harmonics came out correctly in the matrix inversion, suggesting that the matrices are not very well conditioned for the case where the Magsat orbit was approximated. The recursive least squares also performed somewhat better.

Several other methods are available to solve for these coefficients and will be reported on shortly.

	Internal		External		
Harm	Cos	Sin	Cos	Sin	
1	- 35.09	22.04	- 2.92	- 0.56	Input
2	22.28	4.44	1.86	- 5.48	Coefficients
3	- 22.95	26.89	2.29	11.16	
4	18.87	0.20	- 2.61	- 7.75	
5	7.86	27.34	1.74	6.68	
6	- 0.69	17.99	- 1.50	- 0.61	

$a_o = -0.25$

1	- 19.07	11.05	- 19.08	10.78	Recursive
2	11.83	- 0.51	12.13	- 0.83	Least
3	- 10.28	19.06	- 10.42	19.27	Squares
4	7.98	- 2.80	7.81	- 3.81	
5	4.28	16.29	5.27	16.98	
6	- 0.11	15.01	- 1.22	2.06	

$a_o = 3.20$

1	- 27.82	165.92	- 10.67	-146.27	Matrix
2	68.31	- 9.34	- 50.85	7.50	Inversion
3	- 6.40	40.79	- 18.44	79.17	
4	43.78	- 70.01	- 24.00	63.43	
5	10.39	28.49	3.88	7.05	
6	- 0.72	18.02	- 1.50	- 0.65	

r int =r ext = 6800 km

Table 1

Harm	Internal Cos	Internal Sin	External Cos	External Sin	
1	- 35.09	22.04	- 2.92	- 0.56	Input Coefficients
2	22.28	4.44	1.86	- 5.48	
3	- 22.95	26.89	2.29	11.16	
4	18.87	0.20	- 2.61	- 7.75	
5	7.86	27.34	1.74	6.68	
6	- 0.70	17.99	- 1.50	- 0.61	
- 0.00					
1	- 35.07	21.92	- 1.68	1.03	Recursive Least Squares
2	22.24	4.37	0.24	0.04	
3	- 22.88	26.84	- 0.06	0.07	
4	18.80	0.20	0.01	0.00	
5	7.83	27.24	0.00	0.00	
6	- 0.68	17.92	- 0.00	0.00	
- 0.46					
1	- 14.60	- 0.45	-424.72	463.95	Marrix Inversion
2	28.26	12.16	-551.55	-629.67	
3	- 34.26	15.65	3601.09	4608.37	
4	22.91	- 35.98	6193.85	64513.81	
5	9.35	27.59	8247.71	131.75	
6	- 0.70	18.02	- 77.75	-466.09	

r int = 6400 km r ext = 5 x r int

Table 2

Harm	Internal Cos	Internal Sin	External Cos	External Sin	
1	- 35.09	22.04	- 2.92	- 0.56	(Input Coefficients)
2	22.28	4.44	1.86	- 5.48	
3	- 22.95	26.89	2.29	11.16	
4	18.87	0.20	- 2.61	- 7.75	
5	7.86	27.34	1.74	6.68	
6	- 0.69	17.99	- 1.50	- 0.61	
- 0.22					
1	- 20.00	11.64	- 18.97	10.75	(Recursive Least Squares)
2	12.38	- 0.40	12.13	- 0.63	
3	- 10.80	19.68	- 10.45	18.92	
4	8.34	- 2.79	7.96	- 3.65	
5	4.38	16.80	5.19	16.81	
6	- 0.07	13.44	- 1.25	3.76	
1.52					
1	105.00	97.15	-148.91	- 76.98	(Matrix Inversion)
2	- 56.05	54.52	83.15	- 57.60	
3	32.90	40.61	- 57.87	- 4.37	
4	- 12.62	7.99	29.46	- 12.65	
5	7.62	26.21	1.54	5.59	
6	- 0.68	17.99	- 1.52	- 0.61	

r int = 6400 r ext = 7200

Table 3

	Internal		External		
Harm	Cos	Sin	Cos	Sin	
1	- 35.09	22.04	- 2.92	- 0.56	(Input
2	22.28	4.44	1.86	- 5.48	Coefficients)
3	- 22.95	26.89	2.29	11.16	
4	18.87	0.20	- 2.61	- 7.75	
5	7.86	27.34	1.74	6.08	
6	- 0.70	18.00	- 1.50	- 0.61	
- 0.04					
1	- 32.97	20.08	- 9.86	5.87	(Recursive
2	21.98	3.44	3.79	0.54	Least
3	- 22.45	27.62	- 2.17	2.64	Squares)
4	18.58	- 0.19	1.02	- 0.12	
5	7.90	27.40	0.29	0.88	
6	- 0.72	17.91	- 0.06	0.32	
0.18					
1	- 14.59	29.89	- 67.98	- 29.73	(Matrix
2	48.46	34.21	-151.57	-167.97	Inversion)
3	- 61.68	36.55	377.70	- 64.65	
4	- 3.96	- 29.24	412.28	500.39	
5	9.04	26.39	28.25	- 14.12	
6	- 0.68	18.01	- 1.95	1.22	

rint = 6400 r ext = 2 x rint

Table 4

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	Internal		External		
Harm	Cos	Sin	Cos	Sin	
1	10.05	16.47	- 5.41	7.87	(Input Coefficients)
2	- 3.87	2.30	- 2.91	- 4.97	
3	20.93	- 11.23	- 5.37	6.39	
4	1.21	3.63	- 0.58	- 16.30	
5	- 5.25	- 4.04	8.47	2.55	
6	11.23	- 1.36	- 1.48	3.26	
0.02					
1	7.69	17.24	2.31	5.20	(Recursive Least Squares)
2	- 4.18	1.41	- 0.88	0.15	
3	20.15	- 10.50	1.94	- 1.03	
4	1.14	2.67	0.08	0.23	
5	- 4.90	- 3.95	- 0.16	- 0.15	
6	11.15	- 1.27	0.20	- 0.03	
0.00					
1	- 8.30	17.25	56.03	6.94	(Matrix Inversion)
2	- 19.00	3.46	83.01	- 20.25	
3	42.11	- 23.39	-216.52	129.55	
4	- 1.26	21.81	57.37	-332.44	
5	- 5.94	- 4.17	- 9.39	0.22	
6	11.23	- 1.37	- 1.51	3.74	

Table 5

Harm	Internal Cos	Internal Sin	External Cos	External Sin	
1	10.05	16.47	- 5.41	7.87	Input
2	- 3.87	2.30	- 2.91	- 4.97	Coefficients
3	20.93	- 11.23	- 5.37	6.39	
4	1.21	3.63	- 0.58	- 16.30	
5	- 5.25	- 4.04	8.47	2.55	
6	11.23	- 1.36	- 1.48	3.26	

\- 1.70

Harm	Internal Cos	Internal Sin	External Cos	External Sin	
1	2.32	10.41	2.30	9.13	Recursive
2	5.34	4.24	- 4.49	- 3.69	Least
3	13.09	- 3.41	0.73	3.40	Squares
4	- 2.60	- 2.25	- 2.90	- 19.18	
5	- 3.11	- 5.08	7.87	3.42	
6	11.35	- 1.03	- 1.33	3.29	

3.59

Harm	Internal Cos	Internal Sin	External Cos	External Sin	
1	16.42	22.33	- 11.06	2.70	Matrix
2	- 6.66	5.33	- 3.90	- 3.96	Inversion
3	20.02	- 12.03	- 5.20	6.57	
4	1.37	3.45	- 0.55	- 16.33	
5	- 5.23	- 4.02	8.46	2.55	
6	11.23	- 1.36	- 1.48	3.26	

r int = r ext = 6800

Alt = 6800 + 3400, sin θ

Table 6